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Analysis of Dilemma Zone Driver Behavior at Signalized Intersections

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ABSTRACT

This research evaluated the behavior of vehicles between 2.0 and 5.5 seconds upstream of signalized intersections at the start of yellow, as this region is typically considered the "dilemma zone" for drivers. A field study was undertaken in which vehicles were recorded using 8mm analog video cameras temporarily installed at four high-speed intersections (speed limits ≥ 40 mph) and two low-speed intersections (speed limits ≤ 35 mph) in the Madison, Wisconsin area. Approach speed limits at the six study sites ranged between 25 mph to 50 mph. Data were obtained for 1,001 vehicles (463 first-to-stop, 538 last-to-go). Several factors were recorded for each vehicle, including: approach speed, distance upstream of the intersection at start of yellow, brake-response time, average deceleration rate, vehicle type, headway, tailway, action of vehicles in adjacent lanes, presence side-street vehicles/pedestrians/bicycles, presence of opposing vehicles waiting to turn left, flow rate, and cycle length. The research evaluated deceleration rates and brake-response times for first-to-stop vehicles in addition to the differentiating characteristics between first-to-stop and last-to-go vehicles. The observed 15th, 50th, and 85th percentile brake-response times for first-to-stop vehicles were found to be 0.7, 1.0, and 1.6 seconds, respectively. The observed 15th, 50th, and 85th percentile deceleration rates for first-tostop vehicles were 7.2, 9.9, and 12.9 ft/s², respectively. Deceleration rates were found to increase as approach speed increased (i.e., faster drivers used greater deceleration), decrease as distance from the intersection increased (i.e., drivers used lower deceleration when farther from the intersection), and increase as the brake-response time increased (i.e., slower-reacting drivers used greater deceleration rates). The findings suggest that the 10 ft/s² default "comfortable" deceleration rate commonly used for timing yellow intervals may be overly conservative at higher speed intersections as 69 percent of drivers approaching at speeds greater than 40 mph used a deceleration rate greater than 10 ft/s^2 . Drivers were found to be more likely to stop rather than go through under the following conditions: greater travel time to the intersection at start of yellow; shorter yellow-interval; longer cycle length; if the subject vehicle was a passenger vehicle; presence of vehicles/bicycles/pedestrians waiting on the side-street; and absence of vehicles in adjacent lanes that go through. Of these factors, the estimated travel time to the intersection at the start of the yellow interval was found to have, by far, the strongest effect on a driver's likelihood to stop versus go through the intersection.

Key Words: dilemma zone, deceleration, signalized intersection, yellow interval, driver behavior

INTRODUCTION

When a traffic signal changes from a green to a yellow indication, drivers approaching the intersection must quickly decide whether to stop or continue through the intersection. Drivers often experience anxiety when faced with this situation and there are often serious implications when incorrect decisions are made. Drivers who are relatively close to the intersection and abruptly stop when they should proceed, risk rear-end collision from vehicles following close behind. On the other hand, drivers who are farther away from the intersection yet choose to proceed, run the risk of red-light running and a right-angle collision from conflicting vehicles.

Two types of "dilemma zone" situations may arise for drivers when faced with a yellow indication on the approach to a signalized intersection:

- The first type of dilemma zone situation occurs at locations where the yellow and allred times are not of sufficient length, causing a situation where some drivers cannot stop in time for the red indication without uncomfortable braking and also cannot safely clear the intersection without considerable acceleration (1). This situation can be resolved through the use of yellow and all-red intervals that are of sufficient length, based largely on prevailing speeds at the site.
- The second and more common dilemma zone situation occurs as a result of differences in driver behavior. The "indecision zone" is typically defined as the area upstream from the stop line between which 10 percent and 90 percent of the drivers will stop in response to the yellow indication (2). The size and location of the indecision zone varies based on numerous factors, although research has shown the indecision zone typically occurs between 2.5 and 5.5 seconds upstream of the stop line (3).

All references to the term "dilemma zone" in this paper represent the latter situation (i.e., "indecision zone").

Goals and Objectives

The primary goal of the research described herein was to develop comprehensive knowledge of the characteristics of driver behavior in the dilemma zone at signalized intersections. Although past research has evaluated driver behavior during the yellow interval, there are limitations to the current body of literature. Little research on dilemma zone driver behavior has been performed over the past 20 years, although traffic conditions, driver habits, and vehicle characteristics have changed during this time period. Most of the previous studies of driver behavior provide a good blueprint for data collection procedures, although none of the studies included enough predictor variables or considered enough traffic/intersection conditions to produce comprehensive models of dilemma zone behavior. Specific behavioral characteristics for dilemma zone drivers that were of interest to this research included:

- characteristics of first-to-stop versus last-to-go vehicles;
- brake-response times for first-to-stop vehicles; and
- deceleration rates for first-to-stop vehicles.

Literature Review

Past research has investigated drivers' dilemma zone behavior in order to evaluate and characterize stopping characteristics during the yellow and all red intervals. Findings from these

studies have been used to establish default values for certain parameters, such as deceleration rates and brake-response times, used for timing yellow and all-red intervals.

Brake-Response Times for First-to-Stop Vehicles

A driver approaching a signalized intersection at the onset of yellow must first perceive the yellow indication and then decide whether to stop or proceed through. If a driver chooses to stop, he/she must then move his/her foot to the brake pedal and apply enough pressure on the pedal to stop the vehicle. The time between the onset of yellow and the moment the brake light indications are illuminated is referred to as the brake-response time. In cases where the stopping situation does not require immediate braking, the brake-response time may also include a certain amount of additional driver lag time.

In the mid-1980's Chang and Messer, et al., investigated driver response time to the yellow indication for 579 stopping vehicles at six intersections with speed limits between 30 and 50 mph and found that the 15th, 50th, and 85th percentile brake-response times were 0.7, 1.1, and 1.9 seconds, respectively (4). The authors found that increasing approach speed caused a decrease in brake-response time, especially for the 85th percentile response time. At speeds over 40 mph, observed brake-response times were practically unaltered by approach speed or distance from the intersection with a constant median of 0.9 seconds (4). These results were similar to those found in an earlier study by Wortman and Matthais (5). The median values found in the literature provide good agreement with the Institute of Transportation Engineer's (ITE) recommended brake-response time of 1.0 seconds for timing of the yellow interval (6,7). Using data from sixteen driver-response studies, Koppa showed that alerted drivers (typical of drivers approaching a traffic signal) posses mean, 85th, and 95th percentile perception-reaction times for braking of 0.54, 0.64, and 0.72 seconds, respectively (8). Koppa's findings suggest that alerted drivers can react quickly if necessary and confirm that brake-response times determined based on brake-light observations often include additional driver lag time because maximum braking performance in the dilemma zone is typically not necessary.

Deceleration Rate for First-to-Stop Vehicles

After the decision to stop has been made and the driver has moved his/her foot over to the brake pedal, the brakes must be applied with enough pressure to stop the vehicle prior to entering the intersection. ITE recommends that yellow intervals be timed based on a threshold "comfortable" approach deceleration rate of 10 ft/s^2 (6,7). AASHTO suggests a slightly higher threshold deceleration rate of 11.2 ft/s^2 , which most vehicle braking systems and wet tire-pavement friction levels are capable of providing (9). Past research has shown that drivers' selection of deceleration rate in the dilemma zone condition is dependent on approach speed, among other factors.

Chang and Messer determined the 15th, 50th, and 85th percentile deceleration rates for 579 stopping vehicles at six intersections with speed limits between 30 and 50 mph to be 5.6, 9.2, and 13.5 ft/sec², respectively (4). The study also found that as the approach speed increased, the deceleration rate increased at nearly a linear rate suggesting that deceleration rate is highly dependent on approach speed. Selection of deceleration rate was also found to be highly dependent on brake-response time, although neither light condition (day vs. night) nor weather condition (dry vs. wet) were found to have significant effects on deceleration rate. The authors did note that the deceleration rates were based more on selection of comfort and were not indicative of a maximum deceleration rate since it appeared that many of the drivers could have

braked harder if necessary (4). Williams found similar distributions of deceleration rates in his late-1970's study of driver behavior (10). Wortman and Matthais, however, found slightly higher deceleration rates in a mid-1980's study of first-to-stop vehicles at six intersections, finding 15th, 50th, and 85th percentile deceleration rates of 8 ft/s², 11 ft/s², and 16 ft/s², respectively. (5). Koppa showed that alerted drivers stopping for an object in the roadway possessed 5th percentile, 25th percentile, and mean deceleration rates of 8.7, 11.6, and 14.5 ft/s², respectively (8). These findings show that alerted drivers can decelerate at a greater rate than typically observed in dilemma zone stopping situations, suggesting that maximum deceleration performance in the dilemma zone is generally not required.

Probability of Stopping in Time for Red

The probability of a driver stopping or proceeding through the intersection when presented with a yellow indication is largely influenced by the speed and distance (or estimated travel time) of the vehicle from the intersection. Zegeer and Deen performed an analysis of drivers' ability to stop for the red indication (2). At 55 mph, the 10th and 90th percentile stopping distances were 230 and 380 feet upstream of the stop line, respectively. At 45 mph and 35 mph, the 10th and 90th percentile stopping distances were 150 and 315 ft, and 100 and 245 ft, respectively. Wortman and Matthais found similar results in their study of stopping vehicles at higher speed intersections (5).

Williams (10) and later Chang and Messer, et al, (4) performed similar studies of stopping characteristics, although their findings were based on estimated travel time to the stop line rather than distance. Williams' study showed that the 10^{th} , 50^{th} , and 90^{th} percentile time upstream from the intersection for first-stopping vehicles were 1.8, 2.9, and 4.9 seconds, respectively (10). Chang and Messer, et al, found that nearly all vehicles will go through the intersection at 2.0 seconds upstream from the intersection at the start of yellow and 85 and 95 percent of vehicles that went through were less than 3.7 and 4.3 seconds upstream, respectively (4). Eighty-five percent of the vehicles that stopped were greater than 3.0 seconds upstream. These findings were largely insensitive to approach speed.

Chang and Messer, et al, used stepwise logistic regression on their dataset to model drivers' decision to stop or go when presented with a yellow indication as a function of approach speed and distance from the intersection (4). The model showed that the probability of stopping increased as distance from the intersection increased and as approach speed decreased. Distance from the intersection was found to be the predominant predictor of stopping. Validation of the model showed an 80 percent accuracy of the prediction of stopping versus going.

METHODOLOGY

A field study was performed during which vehicles were monitored on the approach to signalized intersections using a video-based data collection system. Data collection focused on recording the behavior of the last vehicle to go through and the first vehicle to stop in each lane during each yellow interval. Data were collected at six different intersections in the Madison, Wisconsin area from April through August, 2006. Sites were selected based on criteria suggested by Bonneson, et al (11). The group of selected sites collectively included:

- approximately level approach grades;
- adequate sight distances;
- approximately 90 degree approach-legs;
- both isolated and closely-spaced signals;

- both coordinated fixed-time and actuated signals;
- varying approach speed limits;
- varying traffic volumes;
- varying cycle lengths;
- varying phase sequences;
- varying clearing widths; and
- varying times for yellow and all-red clearance intervals.

The primary site characteristics for the intersections are shown in Table 1.

	Sites									
Characteristics	Johnson at Park	Verona at Raymond	Verona at McKee	John Nolen at Lakeside	Fish Hatchery at Caddis	East Washington at Baldwin				
Days Recorded	4	2	3	1	2	1				
Total Hours of Video	8	6	14	4	10	4				
Num. of Approach Lanes	4	3	2	3	2	3				
Speed Limit (mph)	25	40	50	45	40	35				
Yellow Duration (sec)	3.5	4.5	5.0	4.0	4.0	3.5				
All-Red Time (sec)	3.0	1.75	2.0	1.5	1.0	1.0				
Intersection Width (ft)	90	90	125	80	90	70				
Signal Actuation	Fixed-time (Coordinated)	Fully Actuated	Fully Actuated	Fully Actuated	Fully Actuated	Fixed-time (Coordinated)				
Development	Urban	Suburban	Suburban	Urban	Suburban	Urban				
Proximity to Upstream Signalized Intersections	0.2 miles	0.7 miles	Isolated	0.8 miles	0.7 miles	0.5 miles				

TABLE 1 Site Characteristics

Data Collection

Vehicles were recorded on the intersection approaches using an 8mm analog video camera mounted on top of a modular steel pole 18 feet in length that was securely strapped to a rigid roadside signpost. The camera mounting system could be installed in as little as 20 minutes and provided two to four hours of continuous, unattended recording. During a given data collection event, all necessary data were recorded at the site using a single video camera. Two similar cameras were used interchangeably, allowing for two sites to be recorded at any given time. One to two recording periods were typically performed for a site during a single day and the camera and mounting poles were removed at the end of the day. Data were only recorded during dry pavement conditions during daylight hours. A typical video camera installation is shown in Figure 1.

The cameras were installed approximately 400 to 800 feet upstream of the intersection and aimed downstream at the intersection so that the rear of vehicles could be visible from at least the upstream edge of the indecision zone (typically 300 to 500 feet from the intersection) continuing all the way through the intersection. From this vantage point, the cameras were afforded full view of the necessary characteristics of the intersection and approaching vehicles, including the traffic signal indication, brake light indications, location of the vehicle with respect to the stop line at any given time, spacing between successive vehicles, and whether or not the vehicle stopped or went through. A general schematic of the field setup for collection of driver behavior data during the yellow interval is shown in Figure 2.



FIGURE 1 Typical video camera installation.

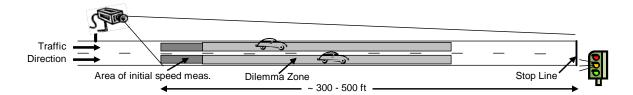


FIGURE 2 Field setup for video recording of driver behavior data.

Data Reduction

Immediately after recording, the 8mm tapes were digitized in preparation for data reduction. Sony Vegas Video 6.0 was used to review and extract the necessary data from the video recordings. Vegas Video allowed frame-by-frame reviewing of the videos to extract the relevant vehicular location and time information. The video was recorded at a rate of 30 frames per second, allowing time to be recorded to the nearest 0.033 seconds. Due to the regular pattern of the white skip-line pavement markings (i.e., 10 foot marking with 30 foot gap), the skip-lines were used as reference markers for determining vehicles' distances upstream from the stop line, which was used at the "zero" reference point at each site. Field measurements of the length of the skip-line pavement markings and the gap between them allowed for a grid to be overlaid onto the computer screen, which provided a scale by which vehicle positioning with respect to the stop bar could be determined to the nearest 5 feet. Figure 3 displays a typical screenshot of a video recording.



FIGURE 3 Screenshot of intersection video.

Data were obtained for "first-to-stop" and "last-to-go" vehicles for each lane for vehicles that were approximately 1.0 to 6.0 seconds upstream at the start of yellow, estimated by dividing the upstream distance by the approach speed. This range of travel times encompassed greater than the 99th percentiles for both stopping and going vehicles. Data were later reviewed and screened prior to the analysis to create a more focused definition of the indecision zone. The following factors were recorded:

- For each "last-to-go" vehicle in each lane:
 - distance from the stop bar at the beginning of the yellow interval;
 - o time elapsed from the onset of yellow until entry to the intersection;
 - o if the vehicle entered after the onset of the red signal indication;
 - distance from the stop bar at the beginning of the red indication and
 - time after the onset of red that entry to the intersection occurred.
- For each "first-to-stop" vehicle in each lane:
 - o distance from the stop bar at the beginning of the yellow interval;
 - o distance from the stop bar when the brake lights become illuminated;

- vehicles whose brake light indications could not be determined (about 4 percent of stopping vehicles) were included in the analysis, although brake-response and deceleration data were left blank;
- o distance from the stop bar when the vehicle stopped;
- time elapsed from the onset of yellow until the brake lights illuminated; and
 time required for the vehicle to stop after the brake lights illuminated.
- For each "last-to-go" and "first-to-stop" vehicle, the following factors were also recorded:
 - o speed immediately before the onset of yellow;
 - time headway with the previous vehicle at the onset of yellow, measured at like points on the vehicles (i.e., front wheel);
 - time tailway with the following vehicle at the onset of yellow, measured at like points on the vehicles;
 - action of vehicles in adjacent lanes than were less than two seconds ahead of the subject vehicle;
 - o presence of vehicles, bicycles, and/or pedestrians waiting on the side-street;
 - o presence of opposing vehicles waiting to turn left;
 - o estimated hourly flow rate per lane;
 - o cycle length; and
 - o vehicle type.
- The following dilemma zone vehicles were excluded from the analysis:
 - o right- and left-turning vehicles;
 - vehicles braking prior to the signal changing to yellow;
 - vehicles approaching during heavily congested conditions (i.e., queuing in through-lanes); and
 - o bicycles.

The distance and travel time information obtained from the videos were used to compute approach speeds, brake-response times, and deceleration rates. Approach speeds were computed using the time to travel between successive grid lines (i.e., 40 or 50 feet) immediately prior to the signal changing to yellow. Brake-response times were computed as the difference between the time at start of yellow and the time when the brake lights illuminated. Occurrences of driver "coasting" (i.e., removing foot from accelerator and not immediately applying the brake) could not be accounted for or quantified based on the data collection methods used here. However, the impact of coasting on the accuracy of deceleration computations was minimized due to the fact that all approaches were on level grades and that vehicular approach speeds were measured immediately prior to the onset of yellow. Deceleration rates were computed as the *average* deceleration rate from the moment the brake lights illuminated to the time when the vehicle stopped using the following formula:

decel rate
$$(ft/s^2) = \frac{approach speed}{braking time}$$
 (1)

The data for the vehicular observations were tabulated, organized, and coded into a single data file for detailed statistical analyses. Initial screening showed that all vehicles less than 2.0 seconds upstream of the intersection at the start of the yellow proceeded through the intersection.

Thus, to create a more concise dilemma zone for the analyses, vehicles that were less than 2.0 seconds upstream of the intersection were removed from the data set. Similarly, with few exceptions, vehicles that were greater than 5.5 seconds upstream at the onset of yellow stopped at the intersection. Thus, the vehicles included in the study were those that were between 2.0 to 5.5 seconds upstream from the stop bar at the onset of yellow. The few cases where vehicles were greater than 5.5 seconds upstream and proceeded through the intersection were included in the data set because each involved a red-light-running event. The travel time boundaries of 2.0 and 5.5 seconds that were used in this analysis were very similar to the 2.5 and 5.5 second boundaries cited in a literature review by Bonneson, et al, as the typical indecision zone boundaries (*3*).

ANALYSIS

After the initial screening, the final data set included data for 1,001 vehicles approaching the intersections during the yellow interval. Each of these 1,001 vehicles was either the first-to-stop (n = 463) or the last-to-go through the intersection (n = 538) after the onset of yellow. The 15th, 50th, and 85th percentile approach speeds for the data used in the analysis were 31, 41, and 51 mph, respectively. To determine any obvious trends in the data, sources for potential bias, and data distributions, the authors initially compared the descriptive statistics (i.e., mean, standard deviation, percentiles, etc.) and simple graphical representations (i.e., histogram, box plot) of the vehicular data on a site-by-site basis. Descriptive statistics for the relevant driver behavior variables are shown in for each site in Table 2.

From there, three primary analyses were performed on the data. The dependent variables for these analyses included:

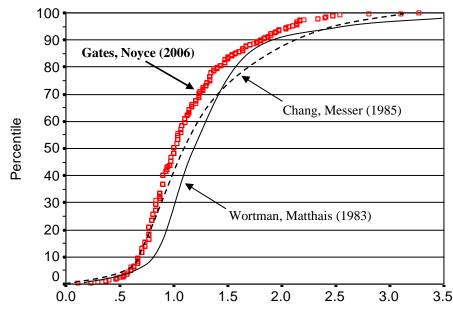
- brake-response time for first-to-stop vehicles,
- deceleration rate for first-to-stop vehicles, and
- probability of a vehicle stopping versus not-stopping.

		Approach Speed Li		Upstrean Line at	Distance Upstream of Stop Line at Start of Yellow (ft)		Estimated Travel Time Upstream of Stop Line at Start of Yellow (sec)		Decel. Rate (ft/s ²)	Flow Rate (veh/hr /lane)	Cycle Length (sec)
Location	Statistic	First to Stop	Last to Go	First to Stop	Last to Go	First to Stop	Last to Go	(sec) First to Stop	First to Stop	/ faile)	(300)
Johnson/ Park	Count	123	59	123	59	123	59	123	123	182	182
	Mean	26.8	28.2	149.4	108.0	3.8	2.7	1.2	8.5	625	96
	S.D.	4.6	5.9	36.2	24.3	0.8	0.6	0.5	2.6	164	14
	15 th %	21.4	21.0	105.0	90.0	3.0	2.1	0.8	6.0	439	80
	85 th %	31.4	34.0	190.0	130.0	4.8	3.3	1.7	11.4	792	110
Verona/ Raymond	Count	81	173	81	173	81	173	79	79	254	254
	Mean	46.6	48.5	312.2	243.7	4.6	3.4	1.2	10.9	503	77
	S.D.	4.6	5.9	45.9	63.6	0.6	0.9	0.5	2.2	100	16
	15 th %	42.6	42.6	265.0	170.5	4.0	2.5	0.8	9.0	413	64
	85 th %	51.1	53.9	355.0	304.5	5.2	4.2	1.7	13.1	595	91
Verona/ McKee	Count	56	37	56	37	56	37	56	56	93	93
	Mean	51.3	52.7	342.1	288.5	4.5	3.7	0.9	11.3	490	127
	S.D.	7.5	6.6	68.7	71.0	0.7	0.8	0.3	2.5	90	14
	15 th %	45.5	45.8	257.8	205.0	3.7	2.9	0.6	8.7	423	115
	85^{th} %	56.8	57.8	389.5	371.5	5.2	4.8	1.1	13.7	606	140
	Count	52	87	52	87	52	87	52	52	139	139
Fish	Mean	40.0	40.2	250.0	177.4	4.3	3.0	1.1	10.1	770	95
Hatchery/	S.D.	4.1	4.8	38.2	36.5	0.6	0.5	0.4	2.5	131	14
Caddis	15^{th} %	35.6	35.6	206.9	141.0	3.7	2.4	0.8	7.9	614	82
	85^{th} %	43.1	45.4	288.3	220.0	5.1	3.5	1.6	12.4	918	110
John Nolen/ Lakeside	Count	61	101	61	101	61	101	57	57	162	162
	Mean	46.9	47.6	280.6	212.2	4.1	3.0	1.2	11.9	473	80
	S.D.	5.9	7.0	51.8	55.6	0.7	0.6	0.5	2.6	125	26
	15^{th} %	40.9	40.8	216.5	151.5	3.3	2.4	0.7	9.7	357	62
	85 th %	51.1	54.4	328.7	271.8	4.8	3.6	1.9	14.3	566	125
East Wash./ Baldwin	Count	90	81	90	81	90	81	77	77	171	171
	Mean	37.4	38.1	232.9	165.8	4.3	3.0	1.0	9.8	357	80
	S.D.	3.1	3.8	40.5	32.6	0.7	0.5	0.4	2.6	30	0
	15^{th} %	34.1	32.7	191.5	135.0	3.5	2.5	0.7	7.3	315	80
	85 th %	40.8	42.4	275.0	205.0	5.1	3.6	1.3	12.2	383	80
TOTAL	Count	463	538	463	538	463	538	444	444	1001	1001
	Mean	39.4	43.5	246.0	203.5	4.2	3.2	1.1	10.1	531	88
	S.D.	10.0	9.0	81.6	70.7	0.8	0.8	0.5	2.8	170	22
	15 th %	27.5	34.1	160.0	135.0	3.4	2.4	0.7	7.2	360	70
	50 th %	40.8	43.8	250.0	195.0	4.3	3.1	1.0	9.9	499	80
	85 th %	51.0	53.8	337.5	278.3	5.1	3.9	1.6	12.9	749	110

Table 2. Descriptive Statistics for Driver Behavior and Stopping Characteristics by Location

Brake-Response Times of First-to-Stop Vehicles

Table 2 shows the overall 15th, 50th and 85th percentile brake-response times for first-to-stop vehicles to be 0.7, 1.0, and 1.6 seconds, respectively. Figure 4 displays the distribution of brake-response times for first-to-stop vehicles both from this study and previous studies.



Brake Response Time (sec)

FIGURE 4. Brake-response times for first-to-stop vehicles.

Figure 4 displays that the data collected here were in good agreement with data collected in previous studies. At lower percentiles (i.e., 60^{th} percentile and lower), the observed brake-response times were very similar to those found by Chang and Messer, et al. (4). At higher percentiles (i.e., 61^{st} percentile and higher), the brake-response times observed here were slightly lower then those observed in previous studies (4,5). The median brake-response time of 1.0 seconds observed here was equal to the default value recommended by ITE for timing of the yellow interval.

The brake-response times were analyzed using univariate multi-factor analysis of covariance (ANCOVA) to investigate the effect of the independent variables. The independent variables included both covariates and categorical factors. The covariates included: approach speed, upstream distance at start of yellow, deceleration rate, estimated hourly flow rate per lane, and cycle length. The categorical factors included: headway (0-2 sec, 2-4 sec, >4 sec), tailway (0-2 sec, 2-4 sec, >4 sec), presence of side-street vehicles/bicycles/peds, presence of opposing left-turning vehicles, action of vehicles in adjacent lanes, and vehicle type (passenger vehicles, heavy vehicles [truck/bus/recreational vehicle (rv)]). The ANCOVA analysis was performed in SPSS v11.5 using the Univariate General Linear Model command (*12*).

Findings

The ANCOVA procedure found brake-response time to be significantly affected (at 95 percent confidence) by approach speed, distance from the intersection at the start of yellow, and deceleration rate. Each of the three significant predictor variables provided approximately equal levels of correlation with brake response time. The other factors – flow rate, cycle length, vehicle type, headway, tailway, presence of side-street traffic, presence of opposing left-turners, action of vehicles in adjacent lanes, and interactions of the categorical variables – were not found to significantly affect brake-response time. The insignificant factors were removed from the analysis and the analysis was re-run to generate the parameter estimates for brake-response time versus approach speed, distance from the intersection, and deceleration rate. The resulting linear regression model for brake-response time for first-to-stop vehicles was estimated as:

brake response time =
$$1.02 - 0.11x_{\text{speed}(mph)} + 0.01x_{\text{dist}(ft)} + 0.18x_{\text{decel rate}(ft/s2)}$$
, $R^2 = 0.548$ (2)

The directions of the parameter estimates in Equation 2 indicate that brake-response time decreased as approach speed increased (i.e., faster drivers reacted more quickly), increased as distance from the intersection increased (i.e., drivers reacted more slowly when farther from the intersection), and increased as the deceleration rate increased.

Deceleration Rates of First-to-Stop Vehicles

Table 2 shows the overall 15^{th} , 50^{th} , and 85^{th} percentile deceleration rates for data reported here to be 7.2, 9.9, and 12.9 ft/s², respectively. Figure 5 displays the distribution of deceleration rates for first-to-stop vehicles from both this study and previous studies.

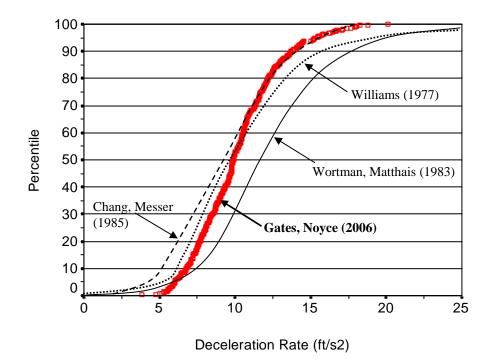


FIGURE 5 Distribution of deceleration rates for first-to-stop vehicles.

Figure 5 displays that the data collected here were in good agreement with data collected in previous studies. At lower percentiles (i.e., 60^{th} percentile and lower), the observed deceleration rates were very similar to those found by Williams (*10*). At higher percentiles (i.e., 61^{st} percentile and higher), the observed decelerations were nearly identical to those found by Chang and Messer, et al. (*4*). The deceleration rates observed by Wortman and Mattahis were higher than the other studies for nearly every percentile (*5*). Note that ITE's recommended comfortable deceleration rate of 10 ft/s² represented the 52nd percentile for the data observed here. It is interesting to note the similarities between the both the deceleration rates and brakeresponse times observed here and those found in previous studies, considering that traffic conditions, vehicle characteristics, and driver behavior have all changed since the previous studies were conducted.

Similar to the brake-response time analysis, deceleration rates were analyzed using ANCOVA to investigate the effect of the independent variables. The covariates included: approach speed, upstream distance at start of yellow, brake-response time, estimated hourly flow rate per lane, and cycle length. The categorical factors included: headway (0-2 sec, 2-4 sec, >4 sec), tailway (0-2 sec, 2-4 sec, >4 sec), presence of side-street vehicles/bicycles/peds, presence of opposing left-turning vehicles, action of vehicles in adjacent lanes, and vehicle type (passenger vehicles, heavy vehicles [truck/bus/rv]). The ANCOVA analysis was performed in SPSS v11.5 using the Univariate General Linear Model command (*12*).

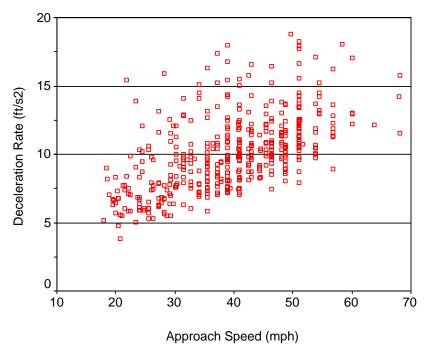
Findings

The ANCOVA procedure found deceleration rate to be significantly affected (at 95 percent confidence) by approach speed, distance from the intersection at the start of yellow, and brake-response time. Of these variables, approach speed was found to have the strongest effect on deceleration rate. The other factors – flow rate, cycle length, vehicle type, headway, tailway, presence of side-street traffic, presence of opposing left-turners, action of vehicles in adjacent lanes, and interactions of the categorical variables – were not found to significantly affect deceleration rate. The insignificant factors were removed from the analysis and the analysis was re-run to generate the parameter estimates for deceleration rate versus approach speed, distance from the intersection, and brake-response time. The resulting regression model for deceleration rate for first-to-stop vehicles was estimated as:

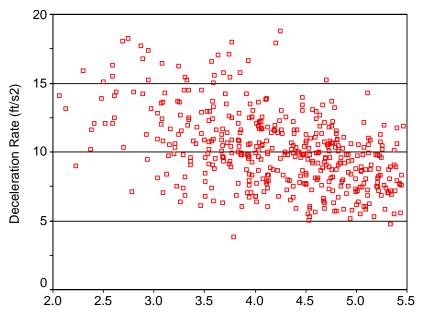
deceleration rate =
$$0.51x_{speed(mph)} - 0.05x_{dist(ft)} + 2.70x_{brake-resp(sec)}$$
, R² = 0.815 (3)

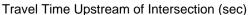
The directions of the parameter estimates in Equation 3 indicate that deceleration rate increased as approach speed increased (i.e., faster drivers used greater deceleration), decreased as distance from the intersection increased (i.e., drivers used lower deceleration when farther from the intersection), and increased as the brake-response time increased (i.e., slower-reacting drivers used greater deceleration rates). The correlations between deceleration rate and the two strongest predictors - approach speed and distance from the intersection (converted to estimated travel time for display purposes) - are shown in Figure 6a-b, respectively. Please note that the estimated travel time was computed by dividing the upstream distance by the approach speed immediately before the onset of yellow and differed from the *actual* travel time to the stop bar, since actual travel times were influenced by any acceleration rate and approach speed and the strong downward trend between deceleration rate and estimated travel time to the intersection at start of yellow. Because of the strong correlations between deceleration and the predictors

approach speed and travel time, the data were split into two approach speed categories (≤ 40 mph, > 40 mph) and two travel time categories ($\leq 4.0 \text{ sec}$, > 4.0 sec) and the deceleration distributions were plotted in Figure 7a-b.



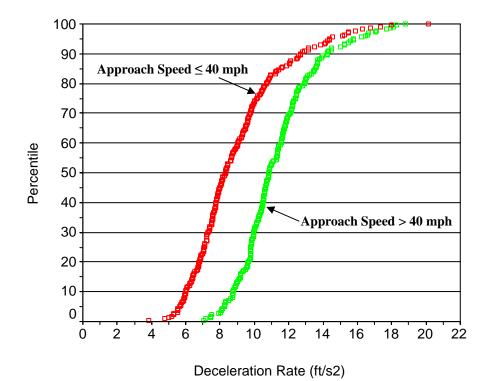
a. Deceleration rate versus approach speed.



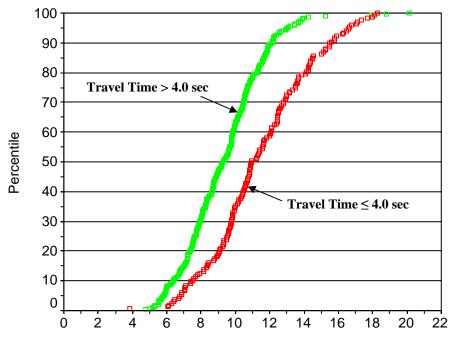


b. Deceleration rate versus estimated travel time to the intersection at start of yellow.

FIGURE 6 Deceleration rate versus speed and estimated travel time to the intersection at start of yellow.



a. Distribution of deceleration rates split by approach speed = 40 mph.





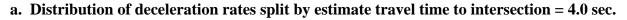


Figure 7. Distribution of deceleration rates split by approach speed and travel time.

Figure 7a shows that drivers approaching at speeds greater than 40 mph will typically use greater deceleration rates than drivers approaching at speeds less than or equal to 40 mph. The 15^{th} , 50^{th} , and 85^{th} percentile deceleration rates for stopping drivers approaching at speeds greater than 40 mph were 9.2, 10.9, and 13.6 ft/s², respectively. The 15^{th} , 50^{th} , and 85^{th} percentile deceleration rates for stopping drivers approaching at speeds of 40 mph or less were 6.4, 8.3, and 11.6 ft/s^2 , respectively. The 10 ft/s² recommended by ITE for comfortable deceleration represented the 31^{st} percentile for speeds greater than 40 mph and 74^{th} percentile for speeds less than or equal to 40 mph for the data observed here. In other words, 69 percent of stopping drivers approaching at speeds greater than 40 mph will use a deceleration rate that is greater than the recommended design value of 10 ft/s². However, only 26 percent of stopping drivers approaching at speeds of 40 mph or less will use a deceleration rate greater than 10 ft/s². These findings suggest that design values for comfortable deceleration at signalized intersections should be based on approach speed rather than a single default value.

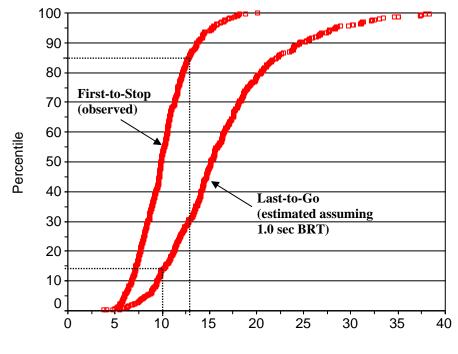
Figure 7b shows that, as expected, drivers with a shorter travel time to the intersection at the start of yellow will typically select a greater deceleration rate than drivers with longer travel times. Half of the stopping drivers who were 4.0 seconds or less of travel time upstream of the intersection used deceleration rates greater than 11.0 ft/s^2 . However, half of the stopping drivers who were less greater than 4.0 seconds upstream used deceleration rates less than 9.3 ft/s². These findings suggest that stopping drivers that are closer to the intersection may not fully compensate by using a quicker brake response time, leading to selection of a greater deceleration rate in order to stop.

Estimated Deceleration Rates for Last-to-Go Vehicles

Further analysis was performed to estimate the deceleration rates necessary for each last-to-go vehicle to stop and subsequently determine the percentage of last-to-go vehicles that were able to comfortably stop. Estimated average deceleration rates for the last-to-go vehicles were computed using the following equation:

decel rate (ft/s²) =
$$\frac{approach speed^2}{2 \times estimated braking distance}$$
 (4)

The braking distance in Equation 4 was estimated as the distance to the stop line from the approximate location that the brake-lights would have appeared assuming a 1.0 second brake-response time. The 1.0 second brake-response time was selected because it represented the median value for the first-to-stop vehicles observed here and is the default design value recommended by ITE. The distribution of estimated average deceleration rates for last-to-go vehicles are shown in Figure 8 along with the distribution of the observed rates for first-to-stop vehicles.



Deceleration Rate (ft/s2)

Figure 8. Distribution of estimated deceleration rates necessary for last-to-go vehicles to stop.

Figure 8 shows that the 85^{th} percentile deceleration rate for first-to-stop vehicles of 12.9 ft/s² was equivalent to approximately the 30^{th} percentile estimated deceleration rate for last-to-go vehicles. Although these findings are purely hypothetical in nature and are sensitive to the assumed brake-response time, Figure 8 suggests that only 30 percent of the last-to-go vehicles could have reasonably been expected to stop. The estimated stopping percentage drops to approximately 15 percent if the ITE threshold comfortable deceleration rate of 10 ft/s² is assumed.

Probability of Stopping versus Going

A forward stepwise binary logistic regression analysis was used to determine the probability of a dilemma zone vehicle to stop or go through at the onset of yellow based on a set of predictor variables. Binary logistic regression is a useful technique for predicting the probability of a dichotomous outcome based on values of a set of predictor variables (continuous or categorical) and is similar to linear regression except that the response variable is categorical rather than numeric. The binary logistic regression model has the form:

$$\ln\left[\frac{p_i}{(1-p_i)}\right] = \alpha + \beta' X_i$$
(5)

- Where: $p_i = \text{Probability}(y_i = y_1 | X_i)$ is the response probability to be modeled (i.e., probability of stopping vs. going based on the set of predictor values), and y_1 is the first ordered level of y,
 - α = Intercept parameter,
 - β' = Vector of slope parameters, and
 - X_i = Vector of predictor variables.

The covariates entered into the model included: estimated travel time to the intersection at start of yellow (based on approach speed and upstream distance), estimated hourly flow rate per lane, cycle length, and yellow time. The estimated travel time was used in the model rather than distance and approach speed since it provided a more universal measure of a vehicle's upstream location with respect to the intersection at the start of yellow. The categorical factors included: headway (0-2 sec, 2-4 sec, >4 sec), tailway (0-2 sec, 2-4 sec, >4 sec), presence of side-street vehicles/bicycles/peds, presence of opposing left-turning vehicles, action of vehicles in adjacent lanes, and vehicle type (passenger vehicles, heavy vehicles [truck/bus/rv]). The binary forward stepwise logistic regression analysis was performed in SPSS v11.5 (*12*). The confidence levels for a predictor to be entered and removed into the forward stepwise model were 0.05 and 0.10, respectively.

The logistic regression analysis found travel time to the intersection, yellow time, action of vehicles in adjacent lanes, vehicle type, presence of side-street vehicles/bicycles/peds, and cycle length to significantly effect whether a vehicle would stop or go through at the onset of yellow. The analysis showed travel time to be, by far, the strongest of the predictor variables. Flow rate, headway, tailway, and presence of opposing left-turning vehicles did not show statistically significant effects on stopping versus going events. Thus, the subsequent probabilities for predicting stopping and going events for dilemma zone vehicles were based solely on the significant predictor variables, using the respective odds-ratio estimates for each. Because the ordinal logistic regression equation is a binary function, only one logistic regression equation was necessary to represent the two responses (stop, go through):

$$\ln\left[\frac{\pi_{go_thru}}{\pi_{stop}}\right] = 2.93 - 2.18x_{trav\ time(sec)} + 1.65x_{yellow(sec)} + .68x_{action\ adjacent} - 1.38x_{veh\ type} + .72x_{side_street} - .01x_{cycle(sec)}$$
(6)

Where: $\pi_{stop} = \text{probability of dilemma zone vehicle stopping in response to the yellow indication;}$ $\pi_{go_thru} = \text{probability of dilemma zone vehicle going through the intersection in response to the}$

Categorical input values are represented by the following:

 $x_{action adjacent} = 1$ for presence of adjacent go-through vehicle(s), 0 for all other cases;

 $x_{veh type} = 1$ for passenger vehicle, 0 for heavy vehicle;

 $x_{side-street} = 1$ for absence of side-street veh/bike/ped, 0 for presence of side-street veh/bike/ped.

Accordingly, based on the preceding equation, the predicted probabilities for each type of event were calculated in the following way:

$$\pi_{go_thru} = \frac{e^{equation(6)}}{1 + e^{equation(6)}} \tag{7}$$

yellow indication;

$$\pi_{stop} = 1 - (\pi_{go \ thru}) \tag{8}$$

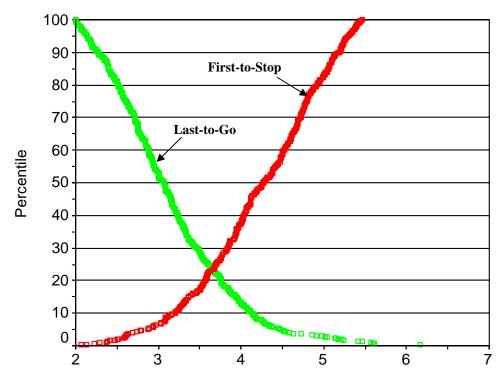
The directions of the parameter estimates in Equation 6 indicated that stopping and going events for dilemma zone vehicles could be predicted based on the following conditions:

- Conditions that were more likely to result in a **stopping** event:
 - o Greater travel time to the intersection at start of yellow;
 - Shorter yellow-interval;
 - Longer cycle length;
 - If the subject vehicle was a passenger vehicle;
 - o Presence of vehicles, bicycles, or pedestrians waiting on the side-street; and
 - Absence of vehicles in adjacent lanes that go through.
- Conditions that were more likely to result in a **go-through** event:
 - Shorter travel time to the intersection at start of yellow;
 - o Longer yellow-interval;
 - Shorter cycle length;
 - o If the subject vehicle was a heavy vehicle (i.e., truck, bus, rv);
 - o Absence of vehicles, bicycles, or pedestrians waiting on the side-street; and
 - Presence of vehicles in adjacent lanes that go through.

Equation 6 had an R^2 value of 0.55 and an overall prediction accuracy of 82 percent for the data observed here. It is important to note that the model was slightly more accurate for predicting go-through events (86 percent correct) versus stopping events (77 percent correct). This provides implications for the use of such a prediction model in interval-extension or phasetermination traffic signal systems. Such systems utilize real-time speed and distance (or travel time) information of approaching vehicles to determine the optimal time to end the green, yellow, or all-red interval to lessen the impacts of red-light-running. With these types of systems, there are greater safety implications associated with inaccurate prediction of go-through events, as go-throughs may lead to red-light running and serious crashes. Thus, the optimal model is one that maximizes the prediction accuracy for go-through events.

Because travel time at the start of yellow was by far the strongest of the individual predictor variables (77 percent total prediction accuracy when used alone), the cumulative percentiles for stopping vs. going *from the actual data set* were plotted versus travel time as shown in Figure 9. Due to the broad range of approach speeds observed in this study, the 15th and 85th percentile probabilities of stopping and going were determined versus travel time and approach speed, with the results shown in Figure 10. The binary logistic regression model for predicting stopping versus going events based solely on travel time to the intersection is shown in Equation 9.

$$\ln\left[\frac{\pi_{go_thru}}{\pi_{stop}}\right] = 6.34 - 1.69 x_{trav\ time(sec)},\ R^2 = 0.42,\tag{9}$$



Travel Time Upstream of Intersection (sec)

FIGURE 9 Distribution of observed estimated upstream travel times at start of yellow for first-to-stop and last-to-go vehicles.

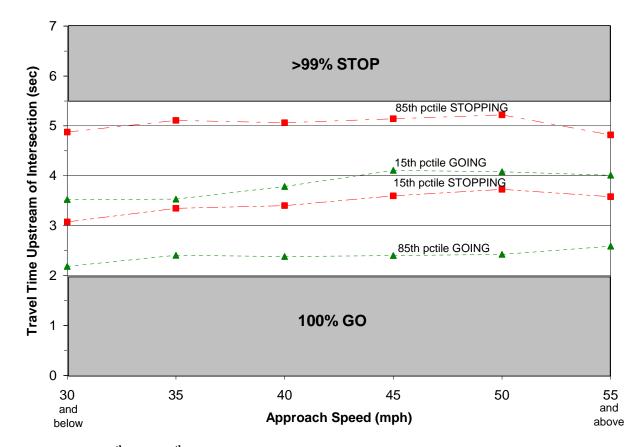


FIGURE 10 15th and 85th percentiles of stopping and going versus estimated travel time and approach speed.

Figures 9 and 10 display a number of important findings. First, no vehicles stopped when less than 2 seconds upstream of the stop line at the start of yellow and only five percent of stopping vehicles were less than 2.9 seconds upstream. The 15th percentile travel time for stopping vehicles was 3.4 seconds upstream at the start of yellow, although the 15th percentile value increased to approximately 3.6 seconds for speeds of 45 mph and above. Less than one percent of going vehicles had travel times greater than 5.5 seconds from the stop line and only five percent of going vehicles were greater than 4.5 seconds upstream. The 15th percentile travel time for going vehicles was 3.9 seconds, although the 15th percentile value decreased to 3.5 seconds for speeds of 35 mph and below. For the most part, approach speed had little to no effect on the 85th percentile travel times, with a slightly upward trend observed for 15th percentile travel times versus increasing approach speeds. These findings were similar to those observed by Chang and Messer, et al, who found that nearly all vehicles will go through the intersection at 2.0 seconds upstream from the intersection and the 5th and 15th percentile travel times for going vehicles were 4.3 and 3.7 seconds, respectively (*4*).

CONCLUSIONS AND RECOMMENDATIONS

The primary goal of this research was to develop comprehensive knowledge of the characteristics of driver behavior in the dilemma zone at signalized intersections. Specific

behavioral characteristics for dilemma zone drivers that were investigated in this research included:

- brake-response times for first-to-stop vehicles;
- deceleration rates for first-to-stop vehicles; and
- characteristics of first-to-stop versus last-to-go vehicles.

Several conclusions and recommendations have been formulated based on the findings and are presented in the following lists along with the summarized findings.

Brake-Response Time

- The overall 15th, 50th, and 85th percentile brake-response times for first-to-stop vehicles were found to be 0.7, 1.0, and 1.6 seconds, respectively.
- Brake-response times were found to decrease as approach speed increased (i.e., faster drivers reacted more quickly), increase as distance from the intersection increased (i.e., drivers reacted more slowly when farther from the intersection), and increase as the deceleration rate increased.
 - Other factors including flow rate, cycle length, vehicle type, headway, tailway, presence of side-street traffic, presence of opposing left-turners, and action of vehicles in adjacent lanes were not found to significantly affect brake-response time.
 - Each of the three significant predictor variables (approach speed, distance from the intersection, and deceleration rate) provided approximately equal levels of correlation with brake response time.
- Because most dilemma zone stopping situations do not require immediate braking, the brake-response times observed here may also include a certain amount of additional driver lag time.
- Similarities were observed between the brake-response times measured here and those found in previous studies conducted several years prior.

Deceleration Rate

- The overall 15^{th} , 50^{th} , and 85^{th} percentile deceleration rates were found to be 7.2, 9.9, and 12.9 ft/s², respectively.
- Deceleration rates were found to increase as approach speed increased (i.e., faster drivers used greater deceleration), decrease as distance from the intersection increased (i.e., drivers used lower deceleration when farther from the intersection), and increase as the brake-response time increased (i.e., slower-reacting drivers used greater deceleration rates).
 - Other factors including flow rate, cycle length, vehicle type, headway, tailway, presence of side-street traffic, presence of opposing left-turners, and action of vehicles in adjacent lanes were not found to significantly affect deceleration rate.
 - Of the three significant predictor variables, approach speed was found to have the strongest effect on deceleration rate.
- Drivers approaching at speeds greater than 40 mph typically used greater deceleration rates than drivers approaching at speeds less than or equal to 40 mph. The 15th, 50th, and 85th percentile deceleration rates for drivers approaching at speeds greater than 40 mph were 9.2, 10.9, and 13.6 ft/s², respectively. The 15th, 50th, and 85th percentile

deceleration rates for drivers approaching at speeds of 40 mph or less were 6.4, 8.3, and 11.6 ft/s², respectively.

- Sixty-nine percent of stopping drivers approaching at speeds greater than 40 mph used a deceleration rate that is greater than 10 ft/s², which is the value specified by ITE as the threshold for comfortable stopping (7). However, only 26 percent of stopping drivers approaching at speeds of 40 mph or less used a deceleration rate greater than 10 ft/s².
 - \circ These findings suggest that the 10 ft/s² default comfortable deceleration rate used for timing yellow intervals may be overly conservative at higher speed intersections.
 - These findings also suggest that design values for comfortable deceleration at signalized intersections should be based on approach speed rather than a single default value.
- Half of the stopping drivers who were 4.0 seconds or less of travel time upstream of the intersection used deceleration rates greater than 11.0 ft/s^2 . However, half of the stopping drivers who were less greater than 4.0 seconds upstream used deceleration rates less than 9.3 ft/s².
 - These findings suggest that stopping drivers that are closer to the intersection may not fully compensate by using a quicker brake response time, leading to selection of a greater deceleration rate in order to stop.
- Approximately 15 to 30 percent of the last-to-go vehicles could have reasonably been able to stop prior to entering the intersection (based on estimated necessary deceleration rates assuming a 1.0 second brake-response time).
- Similarities were observed between the deceleration rates measured here and those found in previous studies conducted several years prior.

Characteristics of First-to-Stop versus Last-to-Go Drivers

- Conditions that were more likely to result in a **stopping** event:
 - Greater travel time to the intersection at start of yellow;
 - Shorter yellow-interval;
 - Longer cycle length;
 - If the subject vehicle was a passenger vehicle;
 - Presence of vehicles, bicycles, or pedestrians waiting on the side-street; and
 - Absence of vehicles in adjacent lanes that go through.
- Conditions that were more likely to result in a **go-through** event:
 - Shorter travel time to the intersection at start of yellow;
 - Longer yellow-interval;
 - Shorter cycle length;
 - o If the subject vehicle was a heavy vehicle (i.e., truck, bus, rv);
 - o Absence of vehicles, bicycles, or pedestrians waiting on the side-street; and
 - Presence of vehicles in adjacent lanes that go through.
- Of the significant predictor variables listed above, the estimated travel time (based on approach speed and upstream distance) was found to have, by far, the strongest effect on drivers' likelihood to stop versus go through.
- A driver's decision to stop or go through can be accurately predicted based solely on the travel time (or speed and distance) upstream of the stop line at the start of yellow.

- No vehicles stopped when less than 2 seconds upstream of the stop line at the start of yellow.
- Less than one percent of going vehicles had estimated travel times greater than 5.5 seconds from the stop line.
- The 5th and 15th percentile estimated travel times for stopping vehicles were 2.9 and 3.4 seconds, respectively.
- The 5th and 15th percentile estimated travel times for going vehicles were 4.5 and 3.9 seconds, respectively.
- Approach speed had little to no effect on the 85th percentile travel times for stopping and going vehicles and only a moderate effect on the 15th percentile travel times.
 - A slightly upward trend was observed for 15th percentile travel times versus increasing approach speeds.
- Similarities were observed between the characteristics of stopping and going events measured here and those found in previous studies conducted several years prior.

Concluding Remarks

The findings shown here provide good agreement with the findings of similar studies conducted several years ago especially considering that traffic conditions, vehicle characteristics, and driver behavior have all changed since the previous studies were conducted. Such findings suggest that driver behavior and performance within the dilemma zone has remained relatively unchanged - perhaps because maximum braking performance in the dilemma zone is typically not necessary, allowing for lag time during brake-response and/or selection of a more comfortable deceleration rate. Drivers that are forced into a more "extreme" dilemma zone condition, such as those approaching at higher speeds and/or at shorter distances from the intersection, were shown to respond by using shorter brake response times and/or greater deceleration rates in order to stop in time. Along those lines, the results of this study suggest that the 10 ft/s² default comfortable deceleration rate commonly used for timing yellow intervals may be overly conservative at higher speed intersections (i.e. approach speeds greater than 40 mph). The authors recommend that design values for comfortable deceleration at signalized intersections be based on approach speed, with greater design deceleration rates used at higher-speed intersections.

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